

Lists

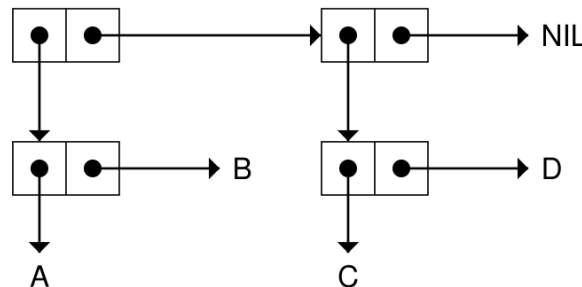
- If a is an atom, then $(e_1 \dots e_n . a)$ represents a data structure that may be drawn as follows:



- A proper list $(e_1 \dots e_n)$ represents a data structure that may be drawn as follows:



$((A . B) (C . D))$ is a *proper list of 2 dotted lists* that represents:



This example is from
p. 73 and p. C-18 of Touretzky.

Further Comments on Dotted Lists

- If a is an atom, then $(e_1 \dots e_n . a)$ represents a data structure that may be drawn as follows:



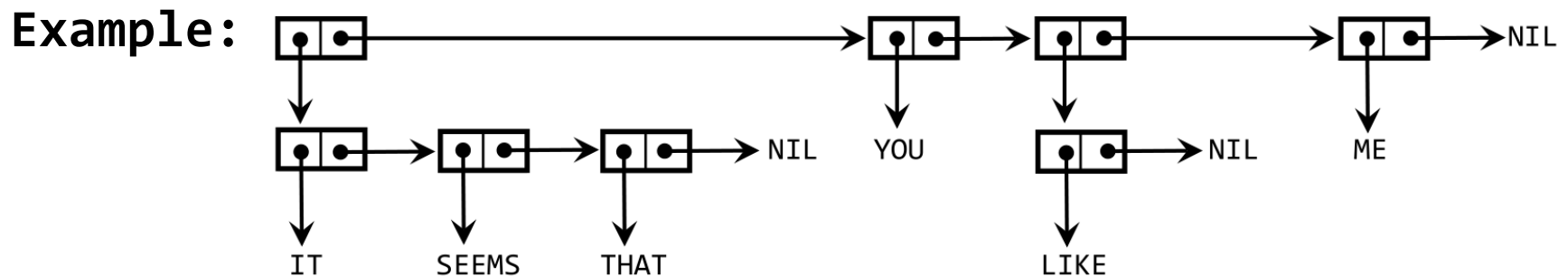
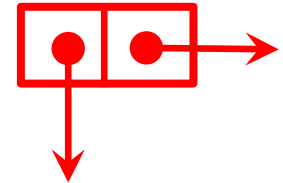
- A proper list $(e_1 \dots e_n)$ represents a data structure that may be drawn as follows:



- Dotted lists are used much less than proper lists.
- The term **list** is often used to mean **proper list**!
- If a function you write for this course returns a dotted list, then either your code has a bug or an inappropriate argument value was passed to the function!

Ways to Draw Cons Cell Representations of Lists

So far we have drawn cons cell representations of lists by depicting each cons cell as a box with left and right parts that represent the *car* and *cdr* fields of the cons cell. (The car and cdr fields store pointers that are depicted as arrows.)



The above is a drawing of the cons cell representation of:

((IT SEEMS THAT) YOU (LIKE) ME)

Note: If you start clisp on mars by entering **cl**, then you can call the **SDRAW** function to produce a drawing of the cons cell representation of a list!

Example of the Use of SDRAW

Welcome to GNU CLISP 2.49 (2010-07-07) <<http://clisp.cons.org/>>

Copyright (c) Bruno Haible, Michael Stoll 1992, 1993

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Type :h and hit Enter for context help.

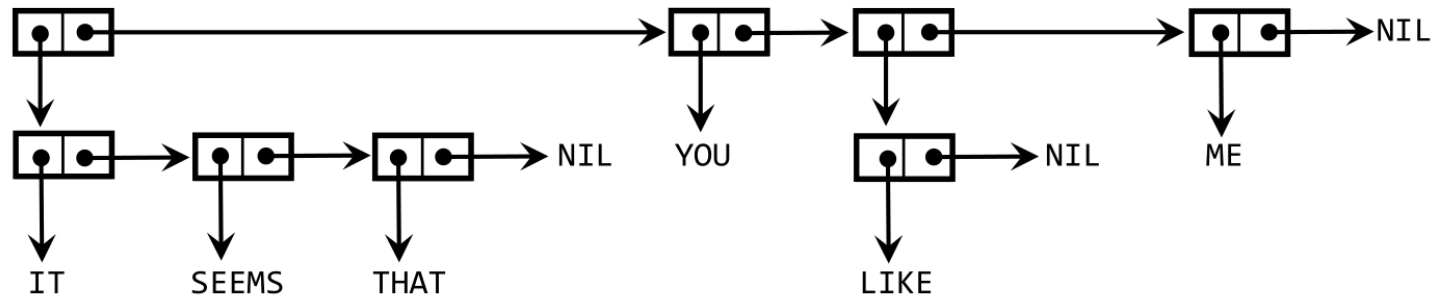
This ' is needed!

[1]> (sdraw '((it seems that) you (like) me))

```
[*|*]----->[*|*]--->[*|*]----->[*|*]--->NIL
|
v
[*|*]--->[*|*]--->[*|*]--->NIL  YOU  [*|*]--->NIL  ME
|           |           |           |
v           v           v           v
IT        SEEMS    THAT        LIKE
```

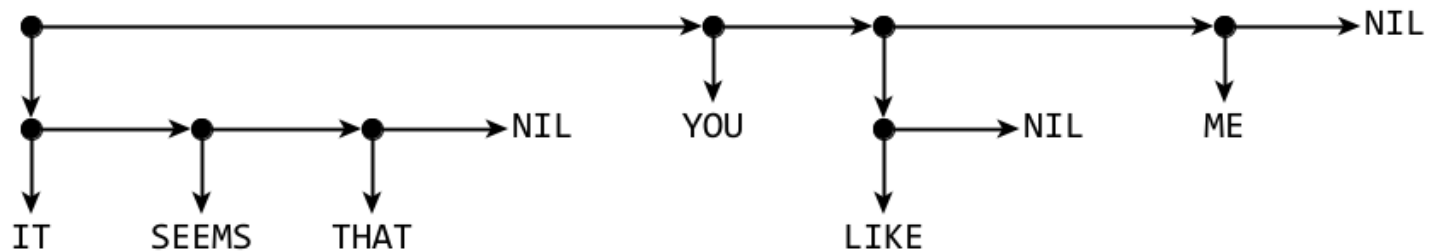
[2]>

Ways to Draw Cons Cell Representations of Lists

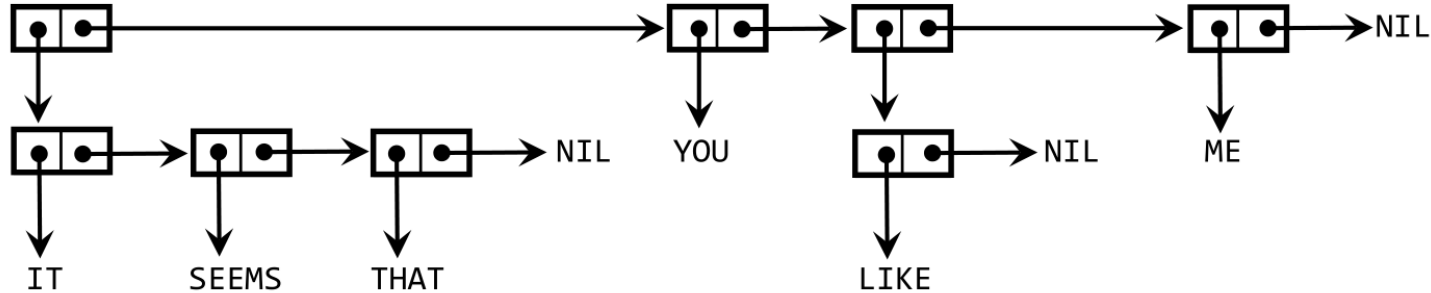


A simpler way to draw a cons cell representation depicts each cons cell as a small dot instead of a box.

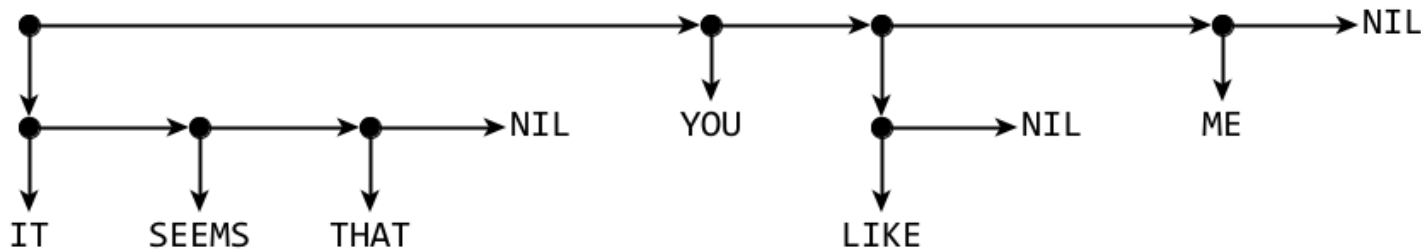
Example The cons cell representation that is drawn above could instead be drawn as follows:



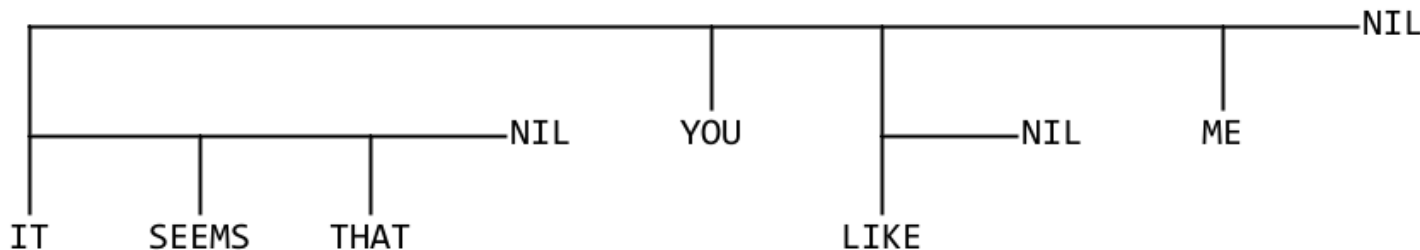
Ways to Draw Cons Cell Representations of Lists



The cons cell representation that is drawn above could instead be drawn as follows:

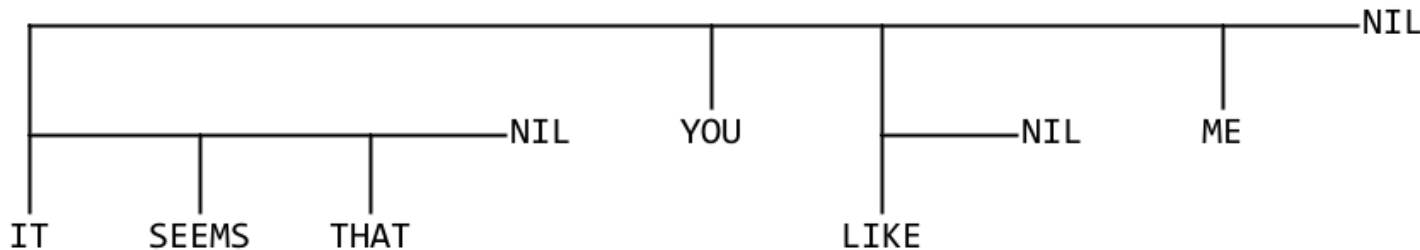


The drawing can be further simplified by **omitting the arrowheads and/or making the dots invisible**. If we do both we get:



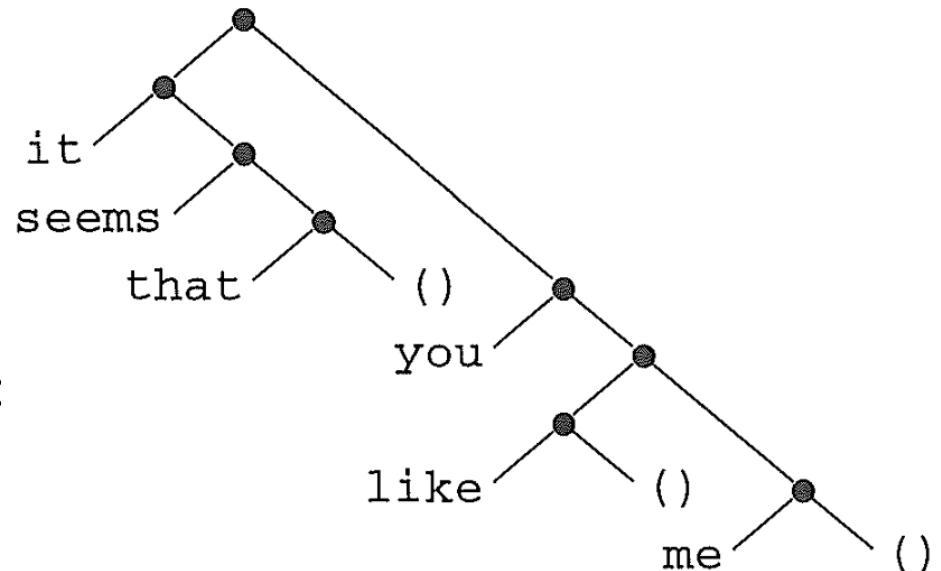
Ways to Draw Cons Cell Representations of Lists

The drawing can be further simplified by **omitting the arrowheads and/or making the dots invisible**. If we do both we get:



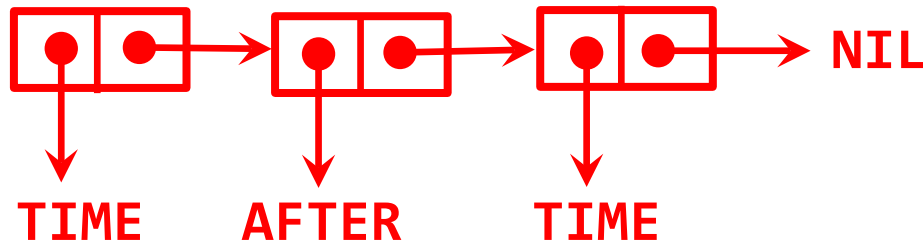
It is also common to draw cons cell representations as binary trees.

For example, the same cons cell representation is drawn on p. 393 of Sethi's book (p. 9 of the course reader) as follows:



Memory Uniqueness of Symbols

(TIME AFTER TIME) represents:



But this drawing needs careful interpretation because the two occurrences of TIME represent *the same identical symbol object*.

Touretzky explains this as follows on p. 195:

In Lisp, symbols are unique, meaning there can be only one symbol in the computer's memory with a given name.** Every object in the memory has a numbered location, called its **address**. Since a symbol exists in only one place in memory, symbols have unique addresses. So in the list (TIME AFTER TIME), the two occurrences of the symbol TIME must refer to the same address. There cannot be two separate symbols named TIME.

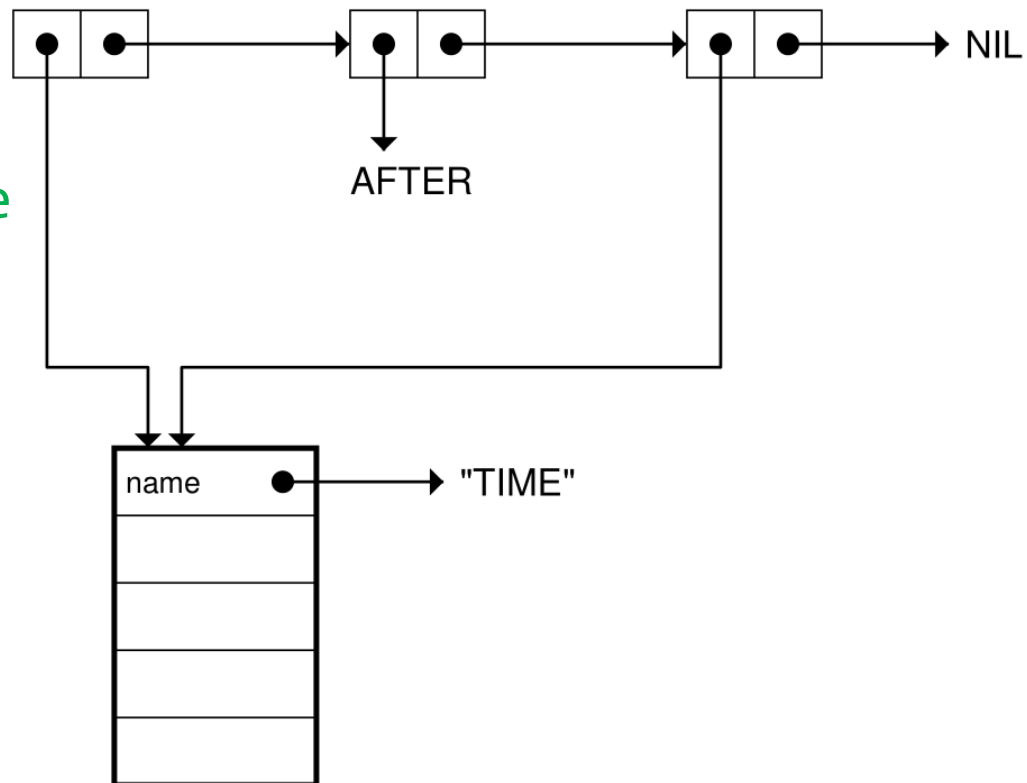
Memory Uniqueness of Symbols

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The following more detailed depiction of the data structure represented by

(TIME AFTER TIME)
is given on p. 196 of Touretzky:

There is *just one*
TIME symbol object!



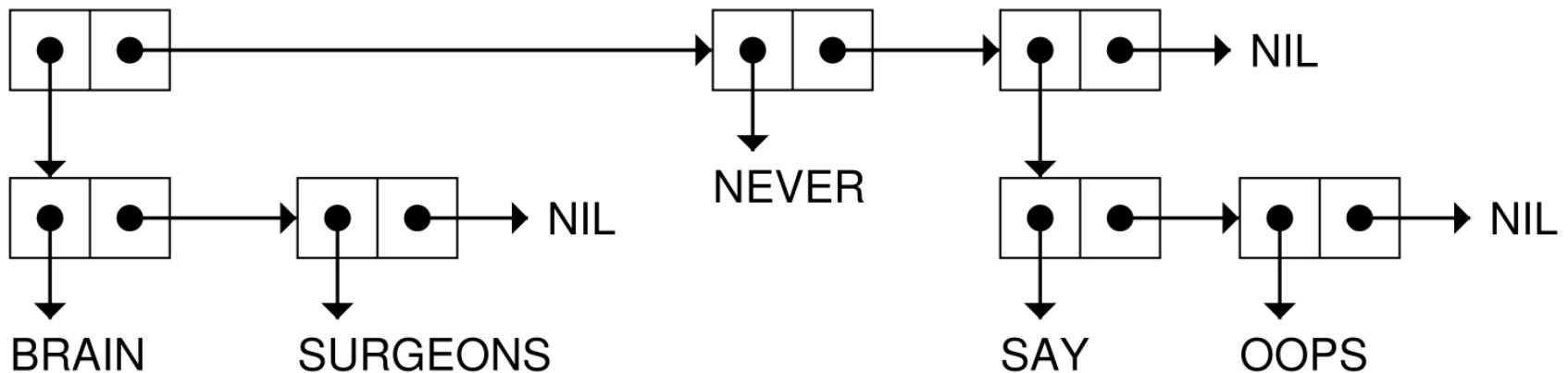
Memory Uniqueness of Symbols

In Lisp, symbols are unique, meaning there can be only one symbol in the computer's memory with a given name.** Every object in the memory has a numbered location, called its **address**. Since a symbol exists in only one place in memory, symbols have unique addresses. So in the list (TIME AFTER TIME), the two occurrences of the symbol TIME must refer to the same address. There cannot be two separate symbols named TIME.

Similarly, all occurrences of the symbol NIL that are shown below represent *the same identical symbol object*:

From p. 34 of Touretzky.

((BRAIN SURGEONS) NEVER (SAY OOPS)) represents:



The QUOTE Special Operator

QUOTE is a special operator you will use most often!

(QUOTE e) evaluates to e; e is not evaluated!

E.g.: The value of **(QUOTE (+ 3 4))** is the list **(+ 3 4)**.

Note: (QUOTE $e_1 \dots e_n$) cannot be evaluated if $n \neq 1$;
evaluation produces an error!

For any S-expression e , ' **e is equivalent to (QUOTE e)**
and we usually write ' e instead of (QUOTE e).

E.g.: We'd write **'(+ 3 4)** instead of **(QUOTE (+ 3 4))**.

If F is the name of some Lisp function, then:

- Evaluation of **(F '(+ 3 4))** passes the list **(+ 3 4)**
as argument to a call of the function F .
- Evaluation of **(F (+ 3 4))** passes the number **7**
as argument to a call of F .
- Evaluation of **(F 'X)** passes the symbol **X**
as argument to a call of F .
- Evaluation of **(F X)** passes the *value of the variable
denoted by X* as argument to a call of F .

- Numbers, NIL, and T evaluate to themselves, so you don't need to QUOTE them!
- As a matter of style, you should not type any spaces between ' and the S-expression e when you type 'e.
- Don't confuse the ` and ' characters:
` is this character! ' is this character!



- ` can do things ' can't. It's useful for writing macros, but you won't need to use ` in this course.

**Built-in
Common Lisp Functions
for Taking Lists Apart:
CAR/FIRST and CDR/REST**

Notation: For S-expressions e and e' , we write
 $e \Rightarrow e'$ to mean that e evaluates to e' .

Examples: $(+ 4 5) \Rightarrow 9$ $(\text{SQRT } (+ 4 5.0)) \Rightarrow 3.0$
 $'(\text{SQRT } (3 \text{ SQRT})) \Rightarrow (\text{SQRT } (3 \text{ SQRT}))$

- If $e \Rightarrow$ a nonempty proper list L , then:
 $(\text{CAR } e) \Rightarrow$ the first element of that list L .

Examples: $(\text{CAR } '(\text{DOG CAT } (\text{AT } (3 +)))) \Rightarrow \text{DOG}$
 $(\text{CAR } '(\text{DOG})) \Rightarrow \text{DOG}$
 $(\text{CAR } '((\text{AT } (3 +)) \text{ DOG CAT})) \Rightarrow (\text{AT } (3 +))$

- If $e \Rightarrow$ a nonempty proper list L , then:
 $(\text{CDR } e) \Rightarrow$ the proper list obtained from L
by omitting its first element.

Examples: $(\text{CDR } '(\text{DOG CAT } (\text{AT } (3 +)))) \Rightarrow (\text{CAT } (\text{AT } (3 +)))$
 $(\text{CDR } '(\text{DOG})) \Rightarrow \text{NIL}$
 $(\text{CDR } '((\text{AT } (3 +)) \text{ DOG CAT})) \Rightarrow (\text{DOG CAT})$
 $(\text{CDR } (\text{CAR } '((\text{A B C}) \text{ D E}))) \Rightarrow (\text{B C})$
 $(\text{CAR } (\text{CDR } '((\text{A B C}) \text{ D E}))) \Rightarrow \text{D}$

- If $e \Rightarrow$ a nonempty proper list L , then:
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Example: $(\text{CAR } '(\text{DOG CAT (AT (3 +))})) \Rightarrow \text{DOG}$

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 $(\text{CDR } e) \Rightarrow$ the proper list obtained from L
by omitting its first element.

Example: $(\text{CDR } '(\text{DOG CAT (AT (3 +))})) \Rightarrow (\text{CAT (AT (3 +))})$

- If $e \Rightarrow$ a nonempty proper list L , then:
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Example: $(\text{CAR } '(\text{DOG CAT (AT (3 +))})) \Rightarrow \text{DOG}$
 - If $e \Rightarrow$ a nonempty proper list L , then:
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by omitting its first element.
Example: $(\text{CDR } '(\text{DOG CAT (AT (3 +))})) \Rightarrow (\text{CAT (AT (3 +))})$
 - $(\text{CAR NIL}) \Rightarrow \text{NIL}$ and $(\text{CDR NIL}) \Rightarrow \text{NIL}$.
This is illogical, but sometimes convenient!
◦ In *Scheme*, the car and cdr of an empty list are undefined.
 - If $e \Rightarrow$ an atom other than NIL , then evaluation of
 $(\text{CAR } e)$ or $(\text{CDR } e)$ will produce an error!
E.g.: We get an error if $(\text{CAR } (+ 3 4))$, $(\text{CDR } (+ 3 4))$,
 $(\text{CAR } (\text{CAR } '(A B)))$, or $(\text{CDR } (\text{CAR } '(A B)))$ is evaluated!
- Q. What happens if $(\text{CAR } '(+ 3 4))$ is evaluated?
What happens if $(\text{CDR } '(+ 3 4))$ is evaluated?

A.

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 $(\text{CAR } (\text{CAR } '(A B)))$, or $(\text{CDR } (\text{CAR } '(A B)))$ is evaluated!
- Q.** What happens if $(\text{CAR } '(+ 3 4))$ is evaluated?
What happens if $(\text{CDR } '(+ 3 4))$ is evaluated?
- A.** $(\text{CAR } '(+ 3 4)) \Rightarrow +$ $(\text{CDR } '(+ 3 4)) \Rightarrow (3 4)$

- If $e \Rightarrow$ a nonempty proper list L , then:
 $(\text{CAR } e) \Rightarrow$ the first element of that list L .
- If $e \Rightarrow$ a nonempty proper list L , then:
 $(\text{CDR } e) \Rightarrow$ the proper list obtained from L
 by omitting its first element.
- $(\text{CAR NIL}) \Rightarrow \text{NIL}$ and $(\text{CDR NIL}) \Rightarrow \text{NIL}$.
- If $e \Rightarrow$ an atom other than NIL , then evaluation of
 $(\text{CAR } e)$ or $(\text{CDR } e)$ will produce an error!

Alternative Names for CAR and CDR

- **FIRST** is another name for **CAR** in Common Lisp.
- **REST** is another name for **CDR** in Common Lisp.

Thus: $(\text{FIRST } e) = (\text{CAR } e)$ $(\text{REST } e) = (\text{CDR } e)$

The names **FIRST** and **REST** have the advantage of being descriptive, but "CAR" and "CDR" provide the basis for the C...R function names we will consider later.

The following remarks regarding the origin of the names **CAR** and **CDR** are from pp. 42 – 3 of Touretzky:

These names are relics from the early days of computing, when Lisp first ran on a machine called the IBM 704. The 704 was so primitive it didn't even have transistors—it used vacuum tubes. Each of its “registers” was divided into several components, two of which were the address portion and the decrement portion. Back then, the name CAR stood for Contents of Address portion of Register, and CDR stood for Contents of Decrement portion of Register. Even though these terms don't apply to modern computer hardware, Common Lisp still uses the acronyms CAR and CDR when referring to cons cells, partly for historical reasons, and partly because these names can be composed to form longer names such as CADR and CDDAR, as you will see shortly.

Here "Register" meant what is usually called a *memory word* today--see the 2nd col. of the 9th page of McCarthy's original paper on Lisp <https://dl.acm.org/doi/pdf/10.1145/367177.367199>.

Also see the "email from Steve Russell" at:
https://www.iwriteiam.nl/HaCAR_CDR.html#Steve

**Built-in
Common Lisp Functions
for Creating Lists:
CONS, LIST, and APPEND**

- If $L \Rightarrow$ a proper list of length n
 then $(\text{CONS } x \ L) \Rightarrow$ a proper list of length $n+1$
 obtained by inserting the value of
 x at the beginning of the list of
 length n .

Examples: $(\text{CONS } 'A \ '(B \ C \ D)) \Rightarrow (A \ B \ C \ D)$
 $(\text{CONS } (+ \ 2 \ 3) \ '(A \ B \ C)) \Rightarrow (5 \ A \ B \ C)$
 $(\text{CONS } '(+ \ 2 \ 3) \ '(A \ B \ C)) \Rightarrow ((+ \ 2 \ 3) \ A \ B \ C)$
 $(\text{CONS } 'A \ \text{nil}) \Rightarrow (A)$

- $(\text{CAR } (\text{CONS } x \ L)) \Rightarrow$ the value of ?
- $(\text{CDR } (\text{CONS } x \ L)) \Rightarrow$ the value of ?

•

.

•

- If $l \Rightarrow$ a proper list of length n
 then $(\text{CONS } x \ l) \Rightarrow$ a proper list of length $n+1$
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 $(\text{CONS } 'A \ \text{nil}) \Rightarrow (A)$

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•

.

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 $(\text{CONS } 'A \ \text{nil}) \Rightarrow (A)$

- $(\text{CAR } (\text{CONS } x \ L)) \Rightarrow$ the value of x
- $(\text{CDR } (\text{CONS } x \ L)) \Rightarrow$ the value of L
- A call of CONS must have exactly two arguments:
Otherwise there will be an evaluation error.
- If the 2nd argument value passed to CONS is not a proper list (i.e., if it is an atom other than NIL or a dotted list), then CONS returns a dotted list. But in this course you are not expected to call CONS this way!

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- A call of CONS must have exactly two arguments:
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The name "CONS" is a shortened form of "construct":

- CONS is the most basic way to construct a new proper list;
but the CDR of that new list will be an existing list.

- $(\text{LIST } e_1 \dots e_k) \Rightarrow$ a proper list of length k whose i^{th} element ($1 \leq i \leq k$) is the value of e_i .

EXAMPLES: $(\text{LIST 'P '(A B) (+ 3 4) '(F)}) \Rightarrow$
 $(\text{LIST (CAR '(A B)) 6 (CDR '(X Y))}) \Rightarrow$
 $(\text{LIST '(U V W)}) \Rightarrow$

- $(\text{LIST } e_1 \dots e_k) \Rightarrow$ a proper list of length k whose i^{th} element ($1 \leq i \leq k$) is the value of e_i .

EXAMPLES: $(\text{LIST 'P '(A B) (+ 3 4) '(F)}) \Rightarrow (\text{P (A B) 7 (F)})$
 $(\text{LIST (CAR '(A B)) 6 (CDR '(X Y))}) \Rightarrow (\text{A 6 (Y)})$
 $(\text{LIST '(U V W)}) \Rightarrow ((\text{U V W}))$